1. Suppose you are a copper tubing manufacturer in Chicago. You know you will need to purchase 50,000 pounds of copper in May and want to hedge the price of copper. You know that the price of copper in Chicago moves closely with the price in New York, and the price differential is due solely to transportation costs from New York to Chicago. Transport costs are variable and will be either \$.03, $\$ .05$ or $\$ .07$ per pound. The current futures price [Ask] for May delivery is $\$ 1.20$ per pound. You decide to hedge using this COMEX copper futures contract.
a. Will this situation be a long or short futures hedge?

Since you are concerned about buying the copper in May sometime out in the future, you want to lock in the price of copper today so this is a long hedge.
b. Is this an inventory or an anticipatory hedge?

Since you are anticipating the purchase of copper in May, this would be an anticipatory hedge strategy.
c. Analyze your hedged costs of copper if the spot price of copper in New York is $\$ 1.000$ or $\$ 1.45$ per pound in May?

Scenario I.

New York Spot Price = \$1.000

| Transport Costs | Copper Costs | Futures Profit | Hedged Costs |
| :--- | :--- | :--- | :--- |
| $\$ .03$ | $50,000 \times 1.03=$ | $50,000(\$ 1.00-\$ 1.20)=$ |  |
|  | $-\$ 51,500$ | $-\$ 10,000$ | $\$ 61,500$ |
| $\$ .05$ | $50,000 \times 1.05=$ | $50,000(\$ 1.00-\$ 1.20)=$ |  |
|  | $-\$ 52,500$ | $-\$ 10,000$ | $\$ 62,500$ |
|  | $50,000 \times 1.07=$ | $50,000(\$ 1.00-\$ 1.20)=$ |  |
|  | $-\$ 53,500$ | $-\$ 10,000$ | $\$ 63,500$ |

Scenario II.

New York Spot Price = \$1.450

| Transport Costs | Copper Costs | Futures Profit | Hedged Costs |
| :--- | :--- | :--- | :--- |
| $\$ .03$ | $50,000 \times 1.48=$ | $50,000(\$ 1.45-\$ 1.20)=$ |  |
|  | $-\$ 74,000$ | $+\$ 12,000$ | $\$ 61,500$ |
| $\$ .05$ | $50,000 \times 1.50=$ | $50,000(\$ 1.45-\$ 1.20)=$ |  |
|  | $-\$ 75,000$ | $+\$ 12,500$ | $\$ 62,500$ |
|  | $50,000 \times 1.52=$ | $50,000(\$ 1.45-\$ 1.20)=$ |  |
|  | $-\$ 76,000$ | $+\$ 12,500$ | $\$ 63,500$ |

Note that this is a perfect hedge --- the hedge costs remain the same regardless of the change in the spot price.
d. How do your hedged costs relate to the current May copper futures price?

If you were in New York, it would be possible for you to lock in $50,000 \times \$ 1.20=\$ 60,000$ in copper costs and not have to deal with the transport costs. However, because you are in Chicago, your costs will be higher. You must add the transportation differentials of:

50,000 x (\$.03) = \$1,500
$50,000 \times(\$ .05)=\$ 2,500$
$50,000 \times(\$ .07)=\$ 3,500$
Thus, your costs are: $50,000 \times \mathrm{F}_{\mathrm{t}, \mathrm{T}}+50,000 \times$ Transport Differential
2. Suppose prices and quantities for the seller of a commodity are as follows:

| Probability | $\mathrm{P}_{\mathrm{T}}$ [Price] | $\mathrm{Q}_{\mathrm{I}}$ [Quantity] |
| :--- | :---: | :---: |
| .25 | $\$ 3.00$ | 120,000 |
| .75 | $\$ 4.00$ | 40,000 |

a. If the beta of the commodity is zero and the CAPM holds, what should the futures price be in equilibrium?

If $\beta$ is 0 , then the futures price should be exactly equal to the expected future spot price.
$F_{t, T}=.25(\$ 3.00)+.75(\$ 4.00)=\$ 3.75$
b. Given the futures price in part a, show what happens in total profits if a hedger enters into short contract equal to the quantity he or she expects to sell.

The expected quantity is: $.25(120,000)+.75(40,000)=60,000$ units

| $\mathrm{P}_{\mathrm{I}}$ | $\underline{Q}_{\underline{I}}$ | $\underline{\text { Revenues }}$ | $\underline{\text { Futures Gain/Loss }}$ | Total Profit |
| :--- | :--- | :--- | :--- | :--- |
| $\$ 3.00$ | 120,000 | $\$ 3.00 \times 120,000=$ | $(3.75-3.00) \times 60,000=$ |  |
|  |  | $\$ 360,000$ | $\$ 45,000$ |  |
| $\$ 4.00$ | 40,000 | $\$ 4.00 \times 120,000=$ | $(3.75-4.00) \times 60,000=$ | $\$ 405,000$ |
|  |  | $\$ 160,000$ | $-\$ 15,000$ | $\$ 145,000$ |

c. Should the optimal hedge ratio be smaller or larger than the expected quantity?

In this case there is an inverse relationship [correlation] between price and quantity [i.e. $\$ 3 \rightarrow \$ 4.00$ against $120,000 \rightarrow 40,000$ ] so you would need to use a hedge ratio smaller than the expected quantity.
d. What hedge will result in zero risk for total profits? Discuss the intuition behind this answer.

What needs to happen is the following:
$\$ 360,000+.75(Q)=160,000-(.25)(Q)$
Total Profits with hedge and the $\$ 3$ price = Total profits with hedge and $\$ 4$ price
$\Rightarrow \mathrm{Q}=-200,000$.

In this case the seller needs to go long 200,000 units. This makes sense because the quantity effect is so great that revenues go down when the price goes up.
3. Suppose the current price of corn is $\$ 4.50$ per bushel, the current futures price for delivery in 3 months is $\$ 4.60$, and this contract is correctly priced. The beta of corn is 0.5 , the expected rate of return on the market is $3 \%$ for 3 months, and the riskless rate of interest is $2 \%$ for 3 months. Assume storage of corn costs $\$ .03$ per bushel per month, paid when the corn is taken out of storage. Is corn a pure asset?

If corn is a pure asset, then the futures market will be at full carry such that:
$F_{t, T}=P_{t}\left(1+r_{t, T}\right)+S C_{t, T}=(\$ 4.50)(1+.02)+3 \times(\$ .03)=\$ 4.68>\$ .4 .60$
In this instance, corn is not a pure asset because the expected futures price is greater than the market futures price for 3 month delivery.

